the channel walls; τ_w, ΔP_w, shear stress and pressure drop in gas suspension flow due to interaction of the particles with the channel walls; τ_s, ΔP_s, shear stress and pressure drop due to weight of particles; τ_f, ΔP_f, shear stress and pressure drop in gas suspension flow; τ_w^e, equilibrium value of shear stress τ_w; w_*^g, w_*^w, w_*^s, w_*^f dynamic velocities calculated from τ_g, τ_w, τ_s, τ_f, respectively; A_0, B_0, parameters of the "law of the wall"; μ_f', flow concentration corresponding to minimum of τ_w^e; ν, ρ, kinematic viscosity and density of carrier medium; d, particle diameter; w_{f1}, particle entrainment velocity; $Re_f = w_{f1}d/\nu$, Reynolds number; y, distance from wall.

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THEORY OF CENTRIFUGAL SEDIMENTATION OF LARGE PARTICLES

Z. I. Abarbanel' and V. S. Kovalenko

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The motion of particles of finite size in rotating viscous liquids is investigated. On the basis of the properties of centrifugal sedimentation described, its possible technical applications are discussed.

In modern science and engineering, centrifugal sedimentation of suspensions containing solid particles of greatly differing sizes is widely used. However, the theory of the motion of particles in a rotating viscous liquid has been developed mainly for fairly small particles [1-4]. The present paper examines the motion of particles of arbitrary size. The results obtained suggest possible new technical applications of the centrifugal sedimentation of suspensions.

Consider the motion of particles of a Stokes diameter a, mass m, and density ρ in a liquid (viscosity η, density ρ_0) rotating at angular velocity ω. The equations of motion of the particles in a coordinate system rigidly fixed in a liquid rotating in a vertical plane are [4]

$$
\begin{aligned}
c\ddot{x} &= sx - b\dot{x} - k\dot{y} + n\sin\omega t,\\
c\ddot{y} &= sy - b\dot{y} + k\dot{x} - n\cos\omega t,
\end{aligned}
\tag{1}
$$

where

$$
s = a^2(\rho-\rho_0)\omega^2,\ b = 18\eta,\ k = 2a^2\rho\omega,
$$
$$
c = a^2\rho,\ n = a^2(\rho-\rho_0)g.
$$

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The initial conditions are

$$x(0)=x_0,\ y(0)=y_0,\ \dot{x}(0)=\dot{x},\ \dot{y}_0(0)=\dot{y}_0.$$

Introducing the complex function z(t) = x(t) + iy(t), the general solution of Eq. (1) is

$$z(t)=Ae^{\gamma_1 t}+Be^{\gamma_2 t}+\frac{in}{c\omega^2-i\omega\mu+s}e^{i\omega t}, \tag{2}$$

where

$$\gamma_{1,2}=\frac{-\mu\pm\sqrt{\mu^2+4cs}}{2c},\ \mu=-b+ik,$$

and A and B are arbitrary constants.

The possibility of analyzing Eq. (2), taking account of the initial conditions in general form, seems slight. Hence, discussion is limited to the motion of particles for times satisfying the condition

$$t\gg\frac{2c}{\alpha+b}=\tau. \tag{3}$$

In this approximation, the particle trajectories are given by the equation

$$z(t)=R_0e^{\gamma t+i(\varphi-\psi_0)}. \tag{4}$$

Here

$$\gamma=\frac{\alpha-b}{2c},\ \varphi=\frac{\beta-k}{2c}t, \tag{5}$$

$$R_0=\frac{x_0}{2}\sqrt{1+\frac{b^2+k^2+2(\alpha b+\beta k)}{\sqrt{(b^2-k^2+4cs)^2+4b^2k^2}}}, \tag{6}$$

while ψ_0 is determined by the initial conditions and the parameters α and β denote the terms

$$\left(\frac{\sqrt{(b^2-k^2+4cs)^2+4b^2k^2}\pm(b^2-k^2+4cs)}{2}\right)^{\frac{1}{2}},$$

where the plus sign corresponds to α and the minus, to β. In writing Eq. (6), it is assumed that $y_0 = 0$. In fact, Eq. (3), which determines the applicability of the solution (4), is adequately satisfied in all practical cases of centrifugal sedimentation. For example, if water of viscosity $\eta = 10^{-2}$ P is used as the sedimentational liquid, then τ is of order 10^{-3} sec for particles of density $\rho = 5$ g/cm^3 and a Stokes diameter on the order of 100 μ, at a rotor angular velocity $\omega = 50$ sec^{-1}. The parameter τ changes significantly with increase in liquid viscosity.

In engineering applications — for example, for granulometric analysis — it is important to know the change in modulus of the radius-vector r of the particle and the angle of rotation φ of the particle with respect to its initial position, in a coordinate system fixed in the rotating liquid.

In the most general case the motion of a particle in a rotating viscous liquid may be represented as follows. In some initial interval of time, the particle moves along a radius-vector with variable linear and angular accelerations. The magnitude and direction of both the linear and the angular accelerations depend on the initial conditions at which the particle motion begins. After a certain interval of time, determined by Eq. (3), the angular acceleration becomes negligibly small and the particle motion is described by the equations

$$r=R_0e^{\gamma t},\ \varphi=\frac{\beta-k}{2c}t. \tag{7}$$

From this moment, the motion may be assumed steady. The parameter R_0 may then be physically interpreted as the level at which steady motion of the particle begins. Numerical estimates for real situations show that R_0 is insignificantly different from the initial level $r_0 = x_0$. In fact, $R_0 - x_0$ is the order-of-magnitude estimate of the particle displacement at time τ. For particles with Stokes diameter a = 100 μ at 50 sec^{-1} and $\theta = \rho_0/\rho = 0.2$, as follows from Eqs. (6) and (7), the displacement is $\delta = 0.03x_0$. For particles of 10 μ and less, it becomes vanishingly small.

Thus, over a broad range of particle sizes it may be assumed, for practical purposes, that R_0 coincides with x_0 and Eq. (7) gives a sufficiently rigorous description of the particle motion.

The main engineering applications of centrifugal sedimentation of particles from suspension are based on the dependence of the precipitation rate of a particle on its Stokes diameter. This dependence is established from Eq. (7) for r and is determined by the parameter γ.

It is interesting to note that at any given moment of time particles of two different sizes may be found at each sedimentation level. This follows from the fact that the positive function $\gamma(a)$ (as $a \to 0$ and $a \to \infty$) tends to zero and has a single maximum. In fact, the expression for γ can be written as follows:

$$\gamma = \omega \sqrt{\frac{1}{2}\left(\sqrt{(\xi-\theta)^2+4\xi}+\xi-\theta\right)} - \omega\sqrt{\xi}, \tag{8}$$

where $\xi = (9\eta/a^2\rho\omega)^2$ is a dimensionless parameter. The extremal point of Eq. (8) is given by the relation

$$2(1-\theta)\xi^3+(8-14\theta+5\theta^2+\theta^3)\xi^2-(2+4\theta-8\theta^2+2\theta^4)\xi+\theta^3(-4+2\theta+2\theta^2)=0. \tag{9}$$

The solution of Eq. (9) is sought in the form of a series

$$\xi = \xi_\varepsilon\theta^\varepsilon + \xi_{\varepsilon'}\theta^{\varepsilon'} + \xi_{\varepsilon''}\theta^{\varepsilon''} + \ldots, \tag{10}$$

where $\varepsilon < \varepsilon' < \varepsilon'' < \ldots$. The undetermined coefficients ε, ε', ε'', ... are eliminated using the Newton diagram [5].

Retaining only the main terms in Eq. (10), the three roots of Eq. (9) are as follows:

$$\xi = -2\theta^3 + o(\theta^3),\quad \xi = -2-\sqrt{5}+o(1),\quad \xi = -2+\sqrt{5}+o(1).$$

The first two roots are negative and are therefore discarded. Determining successive terms of the expansion of the positive root gives

$$\xi_m = 0.236 + 0.83\,\theta + 0.16\,\theta^2 + \ldots. \tag{11}$$

The Stokes diameter of the particles with maximum velocity is

$$a_m = \sqrt{\frac{9\eta}{\rho\omega}}\,\xi_m^{-\frac{1}{4}}. \tag{12}$$

For example, for ρ_0 = 1 g/cm^3, ρ = 10 g/cm^3, $\eta = 10^{-2}$ P, and ω = 50 sec^{-1}, Eq. (12) gives $a_m \approx 178$ μ. By varying the sedimentation parameters the position of the maximum may be regulated and the simultaneous precipitation of particles of two different sizes may be effected.

The second possible technical application of centrifugal sedimentation is based on the relation between the particle size and the angle through which the moving radius vector rotates in the sedimentation time. Eliminating the time t from Eq. (7) gives this dependence in the form

$$\varphi = -\frac{\omega_r}{\omega\sqrt{\xi}}\ln\frac{r}{R_0}, \tag{13}$$

where ω_r is the angular velocity of rotation of the particle in a fixed coordinate system:

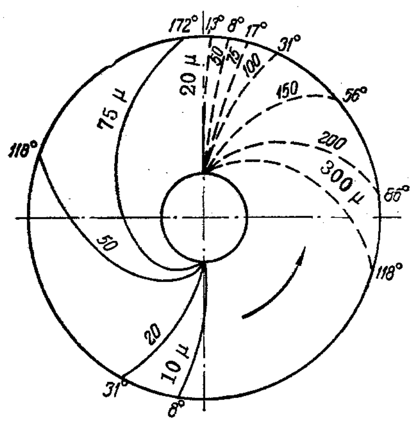

Fig. 1. Trajectories of particles of various sizes. Continuous curves are for ω = 500 sec^{-1} and the dashed curves, for ω = 100 sec^{-1}.

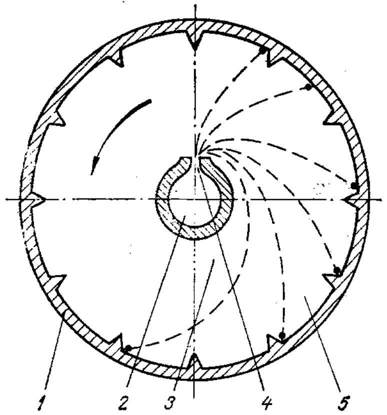

Fig. 2. Centrifugal separation of powders.

$$\omega_r = \omega \sqrt{\frac{1}{2}\left(\sqrt{(\xi-\theta)^2+4\xi}-\xi+\theta\right)}. \tag{14}$$

For the limiting cases of infinitely small and infinitely large particles, Eq. (14) gives $\omega_r = \omega$ and $\omega\sqrt{\theta}$, respectively. Thus, the limits of variation of the angular velocity are

$$\omega \sqrt{\frac{\rho_0}{\rho}} < \omega_r < \omega, \tag{15}$$

i.e., the angular velocity of rotation of the radius-vector of the particle is less than the angular velocity of rotation of the liquid and moves in the same direction. It must be remembered, however, that the limiting estimate for large particles is purely qualitative, since in this case the Stokes law does not apply.

The limiting particle size at which these calculations can be made may be estimated on the basis of the following considerations. The conditions of particle motion are determined by the Reynolds number

$$\mathrm{Re} = \frac{\rho_0 v a}{\eta}. \tag{16}$$

Increase in Re is associated with greater deviation from the Stokes law. For example, as shown in [6] in connection with sedimentational measurements of particle size, the relative error for Re = 0.25, 0.5, and 1.0 was 4.5, 6.5, and 10.5%, respectively.

On the basis of the required accuracy of the results, the acceptable value of Re may be specified, and hence Eq. (16) gives the corresponding limiting value of the particle size. To estimate the limiting particle size, set v in Eq. (16) equal to the particle velocity at the maximum distance R in the radial direction, which may be determined from Eq. (4) in the form

$$v^2 = \frac{\omega^2 R^2}{2\xi}\left(\sqrt{\frac{1}{2}\left(\sqrt{(\xi-\theta)^2+4\xi}+\xi-\theta\right)}-\sqrt{\xi}\right)\left(\sqrt{(\xi-\theta)^2+4\xi}+\xi+\theta\right), \tag{17}$$

where ξ is the dimensionless parameter appearing in Eq. (8). Solving Eq. (16) for the Stokes diameter a and taking Eq. (17) into account, the first approximation for the limiting particle size is

$$a = \left[\frac{18\eta^2 \mathrm{Re}}{\theta(1-\theta)\rho^2\omega^2 R}\right]^{\frac{1}{3}}.$$

For example, for $\eta = 10^{-2}$ P, Re = 0.1, $\rho = 10$ g/cm^3, $\omega = 100$ sec^{-1}, $R = 2 \cdot 10^2$ cm, and $\theta = 0.1$, the limiting particle size $a = 2.16$ μ.

In Fig. 1, trajectories are shown for particles of various sizes precipitating from level R_0 to level $r = eR_0$ at various rates of rotation.

These results for particle motion in a field of centrifugal forces may be used to classify powders by a method illustrated in Fig. 2.

The rotor 1 has two cavities: the internal cavity 2 and the sedimentation cavity 3, which are connected by a narrow slit 4. The rotor begins to rotate and cavity 3 is completely filled with a pure sedimentational liquid, for example, water. Then cavity 2 is filled with a suspension containing powder particles. The centrifugal force drives the particles through slit 4 into cavity 3. The initial radius-vector, determining the particle position in a coordinate system fixed in the rotor, is the same for all the particles.

Since Eq. (13) constitutes a one-to-one relationship between the angle of rotation and the particle size, the particles precipitating in a specific receiving chamber 5 will all belong to a specific fraction, the composition of which is determined by the constructional dimensions of the rotor and may be regulated by appropriate choice of the sedimentational liquid and the angular velocity of the rotor.

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DEVELOPMENT OF A STABLE TWO-PHASE POROUS COOLING SYSTEM

V. A. Maiorov

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The use of a two-layer porous wall instead of a one-layer wall is shown to create the necessary precondition for the practical realization of a two-phase porous cooling system.

The problem of the stationary two-phase porous cooling of a homogeneous plate is formulated in [1, 2]. The properties of this process are investigated and it is established that it is impossible to create a stable two-phase porous cooling system with a homogeneous wall using water as the coolant. The cause of the instability is the substantial reduction in the flow rate of the coolant with a deepening of the zone of vaporization from the outer surface into the porous plate. From this follows the condition for increasing the stability of the system: the variation in the flow rate of the coolant with a deepening of the vaporization zone must be reduced somehow.

An easily realized method of increasing the stability of a two-phase porous cooling system, consisting of using a two-layer porous wall, is proposed and investigated below. The essence of the method becomes clear when an analogy is drawn between the process described

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